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Transfinite Ordinals as Axiom Number Symbols for Unification of Quantum and Electromagnetic Wave Functions

WILLIAM M. HONIG

Western Australian Institute of Technology, Bentley, 6102, *Australia*

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Abstract

The mapping of axioms into transfinite number fields provides a method whereby axioms and the magnitudes of experimental values can be distinguished in a clear manner. This procedure is shown also to result in a logical interpretation for the presence of exponential forms and for their imaginary arguments.

1. *Introduction*

A detailed description of how axioms may be mapped to the arguments of exponentials has been given (Honig, 1976). Such logical interpretations can also be related to physical interpretations for the possible reconciliation of Quantum Mechanics and Electromagnetic Theory including a physical interpretation for h (Honig, 1974). On the basis of these studies one may start directly with the equivalence of the electromagnetic and quantum mechanical wave functions.

2. *Wave Functions and Axioms*

When the electromagnetic wave function, $e^{i\omega t}$, and the quantum mechanical psi function, $e^{iEt/\hbar}$, are set equal to each other:

$$e^{i\omega t} = e^{iEt/\hbar} \tag{2.1}$$

the equality of the arguments results in

$$E = \hbar\omega \tag{2.2}$$

This is the axiom of quantum mechanical and electromagnetic phenomena where E is the energy, ω is the radian frequency, and $\hbar$ is Planck's constant. This procedure also makes it possible to find a number θ that is to stand for the complete equation (2.2). The axiom number symbol, θ, is first given via the correspondence

$$\theta \leftrightarrow (E = \hbar\omega) \tag{2.3}$$

Furthermore, via the mappings

$$\theta \leftrightarrow i\theta \leftrightarrow e^{i\theta} \tag{2.4}$$

these number symbols can give useful interpretations for the relations between electromagnetic and quantum mechanical physical phenomena (Honig, 1976).

We give here an interesting application for transfinite ordinals in this connection which is also a possible relation of these forms [equations (2.1), (2.5), and (2.6)] to continuous sets. The correspondence

$$e^{i\theta} \leftrightarrow e^{i(E=\hbar\omega)} \tag{2.5}$$

is derived from the psi functions and the electromagnetic wave functions by way of the correspondences

$$(e^{iEt/\hbar} = e^{i\omega t}) \leftrightarrow e^{i(E=\hbar\omega)t} \leftrightarrow e^{i\theta t} \tag{2.6}$$

Now if the symbol = in equations (2.2) and (2.5) is replaced by α then the mapping from the first to the second line

$$\begin{matrix} E & = & \hbar\omega \\ \updownarrow & \updownarrow & \updownarrow \\ e^{E} & e^{=} & e^{\hbar\omega} \end{matrix} \tag{2.7}$$

gives for the center mapping the following equation:

$$e^{\alpha} = \alpha \tag{2.8}$$

The solution of this form for α may be considered to be the transfinite ordinal, $\boldsymbol{\omega}$. This is, of course, quite different from ω, the radian frequency which was previously defined. We retain the similarity of terminology because of each term's separate and wide acceptance in mathematics and physics. The $\boldsymbol{\omega}$ transfinite ordinal solution of equation (2.8) seems reasonable because it is well known that $\boldsymbol{\omega}$ does satisfy forms of this kind (Fraenkel, 1961; Fraenkel, 1966; Sierpinski, 1958). Taking account of one of the usual relations for these transfinite ordinals $\exp(\boldsymbol{\omega}) = \boldsymbol{\omega}$ gives the correspondences

$$(E = \hbar\omega) \leftrightarrow (E\boldsymbol{\omega}\hbar\omega) \leftrightarrow (\boldsymbol{\omega}) \leftrightarrow \theta \tag{2.9}$$

The value of θ then becomes a number symbol which as the transfinite ordinal $\boldsymbol{\omega}$ (see references above) possesses the advantage of placing the number field for θ (the complete axiom or equation) beyond the number fields for E, $\hbar$, and ω, which are in the real number field and can be given physical meaning.

3. *Logical Meaning For Exponential Forms*

Some remarks on the set theoretic meaning of the above exponential mappings and forms can be made which relate such matters to the power set axiom. PS, the set of all subsets with the members t, which are discrete, can be conceived of as the insertion set $(p \mid T)$, where p is a pair and T is a set of cardinality t (Fraenkel, 1966), or

$$\mathrm{PS} = 2^t \tag{3.1}$$

Directing one's attention to the value of p; it is equal to 2 (the cardinality of p) for a pair, but this may be conceived of as a way of specifying the discreteness of T. This can be pictured in the mapping diagram (3.2) below where the top line refers to sets (which are identified with the curly brackets) and the lower line with sums of those set members.

$$\begin{array}{ccccccccc} \{1,0\} & \leftrightarrow & \{a,b\} & \leftrightarrow & \{a,b\}^n & \leftrightarrow & \{1,0\} & \searrow & \\ \updownarrow & & \updownarrow & & \updownarrow & & \updownarrow & & 2 \\ (1+0) & \leftrightarrow & (a+b) & \leftrightarrow & (a+b)^n & \leftrightarrow & (1+0) & \nearrow & \end{array} \tag{3.2}$$

The vertical mappings are performed via the correspondences

$$\begin{array}{cc} \{\ \ \} & , \\ \updownarrow & \updownarrow \\ (\ \ ) & + \end{array} \tag{3.3}$$

where cross products of the expansion $(a + b)^n$ are zero for $a = 1$ and $b = 0$. Here 1 corresponds to the presence of a member of the set T, and 0 corresponds to its absence. The item $\{a, b\}^n$ is the Cartesian product set. The variable n is an arbitrarily large integer corresponding to the carrying out of the product operation for an arbitrarily large number of times.

It is tempting, in view of the analysis of Robinson (1966); to view the continuous sets in a similar manner. Instead of $\{1, 0\}$ one takes as the insertion sets $\{1, \Delta\}$ where Δ is an infinitesimal defined as

$$\Delta = \lim(1/n) \quad \text{for} \quad n \to \infty \tag{3.4}$$

Here 1 and Δ are, for example, the points on a unit interval and differential increments from that point along the same interval. One may then repeat the mappings (3.3), and the essential mappings similar to (3.2) are

$$\begin{array}{ccccccc} \{1, 1/n\} & \leftrightarrow & \{1, 1/n\}^n & \leftrightarrow & \{e\} & \searrow & \\ \updownarrow & & \updownarrow & & \updownarrow & & e \\ (1 + 1/n) & \leftrightarrow & (1 + 1/n)^n & \leftrightarrow & (e) & \nearrow & \end{array} \tag{3.5}$$

The vertical correspondence $\{e\} \leftrightarrow (e)$ shows that the insertion set cardinal is e. The final set above is $\{e\}$ and not $\{e, 0\}$ because the null set is ruled out of $\{1, 1/n\}$ by its definition. The cardinality of a continuous set T', therefore, can then be given in a similar manner to equation (3.1) as

$$\mathrm{PS} = e^{t'} \tag{3.6}$$

where t' is the cardinality of T'.

It seems plausible, therefore, to view the form $e^{i\theta}$ on the basis of these ideas as the set of all subsets (PS) to which θ applies, where θ is a *continuous* variable. For the example given here this means that a continuous variation in ω

will give a continuous variation in E. In this way $e^{i\theta}$ will then include all cases to which $E = \hbar\omega$ applies.

4. *Logical Meaning for Imaginary Arguments*

The imaginary term i was not discussed but its logical import is to label axioms. That is, it is to be used to label statements or equations whose mechanism is unknown (Honig, 1976). This can be pictured clearly by including it in the mappings of (2.7) as follows:

$$\begin{array}{llll} \text{Axiom} & & & \\ \updownarrow & & & \\ i & E & = & \hbar\omega \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ e^{i} & e^{E} & e^{=} & e^{\hbar\omega} \end{array}$$

and where the operation which carries the middle line, $iE = \hbar\omega$, to the lowest line is the finding of the cardinality of the set of all subsets to which this middle line applies. The lowest line then maps to the forms (2.5), see Honig, (1976).

Thus, here the logical meaning of i is to label statements whose mechanism is unknown (i.e., axioms) and this is mapped to the fact that the meaning of i (its value) is unknown. This is similar to the Frege procedure of mapping the meaning of paradox (or contradiction) to the meaning of zero.

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